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*Morphic sequences: characterization, visualization and equality*

**Abstract:** Morphic sequences form a natural class of infinite sequences, most times defined by fixed points of morphisms. They cover well-known examples like the Thue-Morse sequence and the Fibonacci sequence. The recursive structure of such a morphic sequence is closely related to a dynamic radix enumeration system for the natural numbers.

In this talk we focus on the following three aspects of morphic sequences:

- (1) Equivalent characterizations of the class of morphic sequences. These include characterizations based on automata, by finiteness of a particular class of subsequences, and by rationality of infinite terms. Some of these extend similar well-known characterizations of automatic sequences.
- (2) Visualization by turtle graphics. Criteria have been developed resulting in either finite turtle figures with a lot of symmetry, or turtle figures with fractal patterns. Playing around with examples satisfying these criteria yield a wide range of amazing figures, each generated by a computer program of just a few lines.
- (3) Proving that different representations define the same morphic sequence. Surprisingly, exploiting a suitable criterion in many cases this can be done fully automatically by a simple computer program. This yields elementary induction proofs, however being quite complex by distinguishing several cases, while checking by hand is not needed as they are correct by construction.

All issues will be illustrated by numerous examples.

