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Mahler equations for Zeckendorf numeration

Abstract: Let $U = (u_n)_{n\geq 0}$ be a Pisot numeration system. A sequence (f_n) taking values over a commutative ring R, possibly infinite, is said to be U*-regular* if there exists a *weighted* automaton which outputs f_n when it reads $(n)_U$. For base-q numeration, with $q \in \mathbb{N}$, q-regular sequences were introduced and studied by Allouche and Shallit, and they are a generalisation of q-automatic sequences (f_n), where f_n is the output of a deterministic automaton when it reads $(n)_q$. Becker, and also Dumas, made the connection between q-regular sequences, and q*-Mahler type* equations. In particular a q-regular sequence gives a solution to an equation of q-Mahler type, and conversely, the solution of an *isolating*, or Becker, equation of q-Mahler type is q-regular.

We define generalised equations of Z-Mahler type, based on the Zeckendorf numeration system Z. We show that if a sequence over a commutative ring is Z-regular, then it is the sequence of coefficients of a series which is a solution of a Z-Mahler equation. Conversely, if the Z-Mahler equation is isolating, then its solutions define Z-regular sequences. We provide an example to show that there exist non-isolating Z-Mahler equations whose solutions do not define Z-regular sequences. Our proof yields a new construction of weighted automata that generate classical q-regular sequences.

This is joint work with Olivier Carton.