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Base-$\frac{p}{Q}$ structure of states in automata arising from Christol's theorem

Abstract: A celebrated theorem of Christol states that a sequence $a(n)_{n\geq 0}$ of elements in the finite field $\mathbb{F}_q$ is algebraic if and only if it is q-automatic. One direction of Christol's theorem was generalized by Denef and Lipshitz as follows. If a sequence $a(n)_{n\geq 0}$ of integers is algebraic then, for every $\alpha \geq 1$, the sequence $(a(n) \bmod p^\alpha)_{n\geq 0}$ is p-automatic. We are interested in the size of the minimal automaton for this sequence (measured as the number of states). Experiments suggest that the size grows exponentially as a function of α, but the bound obtained from the construction is doubly exponential. To reconcile this discrepancy, we describe a new base-$\frac{p}{Q}$ numeration system, where $Q \in (\mathbb{Z}/p^\alpha\mathbb{Z})[x, y]$ and where the digits belong to $(\mathbb{Z}/p\mathbb{Z})[x, y]$. We show that each state in the automaton has a unique representation in this numeration system and, moreover, that the degrees of the digits are bounded. This restricted structure allows us to obtain a singly exponential bound on the number of states.

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