Pitch and Poster Session

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Abstracts

Sohail Farhangi
University of Adam Mickiewicz

Cantor series for which normality and distribution normality coincide

Abstract: A natural generalization of the notion of base b expansions is to fix a **basic sequence** $Q = (q_n)_{n=1}^{\infty}$ with $q_n \geq 2$, and consider the[1] base Q expansion of $y \in [0,1]$ given by

$$y = \frac{a_1}{q_1} + \frac{a_2}{q_1 q_2} + \frac{a_3}{q_1 q_2 q_3} + \cdots = \sum_{n=1}^{\infty} \frac{a_n}{\prod_{i=1}^{n} q_i} = 0.a_1 a_2 \cdots a_n \cdots_Q \,, \tag{1}$$

with $0 \leq a_i < q_i$. The base b expansions correspond to the case in which Q is the constant sequence $(b)_{n=1}^{\infty}$. There are two natural notions of normality that we can associate to a basic sequence. We say that $y \in [0,1]$ is **Q-normal** if every block of digits appearing in the base Q expansion of y appears with the correct frequency (which we do not define precisely here), and we denote the set of such y by $\mathcal{N}(Q)$. We say that $y \in [0,1]$ is **Q-distribution normal** if the sequence $(x \prod_{i=1}^{n} q_n)_{n=1}^{\infty}$ is uniformly distributed in $[0,1]$, and we denote the class of such y by $\mathcal{DN}(Q)$. In contrast to the situation of base b, there exists basic sequences for which $\mathcal{N}(Q) \neq \mathcal{DN}(Q)$. It is therefore natural to try and find classes of Q for which $\mathcal{N}(Q) = \mathcal{DN}(Q)$, which is the goal of the present work. We remark that this is only the beginning of a larger program in which we try to find Q for which the theory of base b normality extends to a theory of normality base Q.

Joint work with Bill Mance.

[1] The expansion is called a Cantor series and is unique for all but countably many $y \in [0,1]$.

Peej Ingarfield
University of Manchester

Thermodynamic Formalism of Self Similar Overlapping Measures

Abstract: A key question in Fractal geometry is to understand the Hausdorff dimension of sets or measures. In recent years there has been progress made to understand more complicated fractal structures, in particular overlapping self-similar measures. There has been focus on the study of Bernoulli convolutions, which lead to overlapping fractals with a variable contraction rate. Rather than studying these we shall be looking at orthogonal projections of the uniform measure on the Sierpinski triangle. These projections form overlapping self similar sets with a variable translation parameter. The main result of this work is to make a connection between dimension theory of these IFSs and thermodynamic formalism of the doubling map restricted to rational slices of the torus. Of note is how we establish a correspondence between the varying transnational parameter and varying rational slices.

Pascal Jelinek
Montanuniversität Leoben

Collisions of digit sums in two bases

Abstract: The problem of the infinitude of numbers that have the same sum of digits in multiple coprime bases, we call such numbers collisions, has been open for many decades. Only recently, in 2023, Spiegelhofer managed to prove a lower bound on collisions in bases 2 and 3, and hence showed for the first time that there are infinitely many such numbers. His methods rely on the following two properties of the sum of digits function in arithmetic progressions of step size $d < N^{1/2-\varepsilon}$.
Firstly the sum of digits of these numbers needs to be concentrated around the expected value and secondly the sum of digits needs to be equidistributed modulo m, where $m \leq \sqrt{\log(N)/\log(\log(N))}$.
In this talk, we will show how both properties can be derived for d of size at most $N^{1-\varepsilon}$, under some mild assumptions on d. Using this, we can find a lower bound on collisions in coprime bases p and q, and hence show that there are infinitely many such numbers.

Savinien Kreczman
Université de Liège

Confluent alternate numeration systems

Abstract: We study normalisation as a rewriting system in the framework of two-way alternate bases.

In this framework, nonnegative real numbers are represented by right-infinite words with a decimal point. A number may have multiple representations, among which one is distinguished unsing a greedy algorithm and called the expansion. The problem of normalisation is to find, given a word, the expansion of the number it represents.

We study this problem by seeing normalisation as a rewriting system, taking a representation as input and iteratively rewriting factors which cannot appear in expansions while keeping the value of the word constant, until the expansion is reached.

We are especially interested in the numeration systems for which this associated rewriting system is confluent, that is, two words that can be obtained by rewriting a common start word can in turn be rewritten as a common end word. We obtain a characterisation of those systems: up to technicalities, all but the last digit in any expansion of 1 must be maximal.

A connection is made to the problem of the equality between the spectrum and the integers of a numeration system. This problem asks whether all numbers that admit a representation only to the left of the fractional point also have their expansion only to the left of the fractional point. Using a similar framework of

rewriting rules, we find a class of systems where this equality is reached, with a criterion similar in statement to the one mentioned above.

Joint work with Émilie Charlier, Zuzana Masáková and Edita Pelantová.

Yao-Qiang Li
Guangdong University of Technology

Expansions of generalized Thue-Morse numbers

Abstract: We introduce generalized Thue-Morse numbers of the form

$$\pi_\beta(\theta) := \sum_{n=1}^{\infty} \frac{\theta_n}{\beta^n}$$

where $\beta \in (1, m+1]$ with $m \in \mathbb{N}$ and $\theta = (\theta_n)_{n\geq 1} \in \{0, 1, \cdots, m\}^{\mathbb{N}}$ is a generalized Thue-Morse sequence previously studied by many authors in different terms. This is a natural generalization of the classical Thue-Morse number $\sum_{n=1}^{\infty} \frac{t_n}{2^n}$ where $(t_n)_{n\geq 0}$ is the well-known Thue-Morse sequence $01101001\cdots$. We study when θ would be the unique, greedy, lazy, quasi-greedy and quasi-lazy β-expansions of $\pi_\beta(\theta)$, and generalize a result given by Kong and Li in 2015. In particular we deduce that the shifted Thue-Morse sequence $(t_n)_{n\geq 1}$ is the unique β-expansion of $\sum_{n=1}^{\infty} \frac{t_n}{\beta^n}$ if and only if it is the greedy expansion, if and only if it is the lazy expansion, if and only if it is the quasi-greedy expansion, if and only if it is the quasi-lazy expansion, and if and only if β is no less than the Komornik-Loreti constant.

Yun Sun
South China University of Technology

Fiber denseness of intermediate β-shifts of finite type

Abstract: We focus on $T_{\beta,\alpha}(x) = \beta x + \alpha \pmod 1$, $x \in [0,1]$ and $(\beta, \alpha) \in \Delta := \{(\beta, \alpha) \in \mathbf{R}^2 : \beta \in (1,2) \text{ and } 0 < \alpha < 2 - \beta\}$. The $T^{\pm}_{\beta,\alpha}$-expansions $\tau^{\pm}_{\beta,\alpha}(x)$ of critical point $c_{\beta,\alpha} = \frac{1-\alpha}{\beta}$ are denoted as (k_+, k_-). Let $\Delta(k_+) := \{(\beta, \alpha) \in \Delta : \tau^{+}_{\beta,\alpha}(c_{\beta,\alpha}) = k_+\}$ with k_+ being periodic, we state that $\Delta(k_+)$ is a smooth curve which can be regarded as a fiber. We extend the results of Parry (1960) and show that, the set of (β, α) with its $\Omega_{\beta,\alpha}$ being a SFT is dense in $\Delta(k_+)$. Similarly for the fiber $\Delta(k_-)$.

When considering another fiber $\Delta(\beta) := \{(\beta, \alpha) \in \Delta : \beta \in (1,2) \text{ is fixed}\}$, we demonstrate that when β is not a multinacci number, there are only countably many distinct matching intervals on $\Delta(\beta)$. We prove that the set of (β, α) with $\Omega_{\beta,\alpha}$ being a SFT is dense in each matching interval.

Joint work with Bing Li and Yiming Ding.

Magdaléna Tinková
Czech Technical University in Prague

Non-decomposable quadratic forms over totally real fields

Abstract: Let K be a totally real field. In this talk, we will consider n-ary quadratic forms $Q(x_1, \ldots, x_n) = \sum_{i,j=1}^{n} a_{ij} x_i x_j$ where coefficients a_{ij} belong to the ring of algebraic integers $\mathcal{O}_K$ of K. We say that Q is totally positive semi-definite if $Q(\gamma_1, \ldots, \gamma_n) \in \mathcal{O}_K^+ \cup \{0\}$ for all $\gamma_i \in \mathcal{O}_K$, where $\mathcal{O}_K^+$ denotes the set of totally positive elements in $\mathcal{O}_K$, i.e., of those elements whose all conjugates are positive. Moreover, our quadratic form is non-decomposable if it cannot be written as $Q = Q_1 + Q_2$ where Q_1 and Q_2 are totally positive semi-definite quadratic forms.

So far, the research on non-decomposable quadratic forms mostly focused on the case when $K = \mathbb{Q}$. For general totally real fields, Baeza and Icaza found a constant C_{BI} satisfying the following. If the algebraic norm of the determinant $N(\det(Q))$ of Q is greater than C_{BI}, then Q can be written as $Q = L^2 + H$ where L is a linear form in n variables and H is a totally positive definite form, which implies that Q is decomposable. In this talk, we will show that their result can be improved considering decompositions of the form $Q = \alpha L^2 + H$ where $\alpha \in \mathcal{O}_K$ is totally positive. Moreover, we will discuss some related results.

Joint work with Pavlo Yatsyna.

Hichem Zouari
Institut Élie Cartan de Lorraine (IECL)

Mean values of some additive arithmetical functions over friable integers

Abstract: Let $S(x,y)$ be the set of integers up to x, all of whose prime factors are $\leq y$, and $s_q(n)$ be the sum-of-digits function in base $(q \geq 2)$ of the positive integer n. Our main result is to estimate the sum $\sum_{n \in S(x,y)} v(n)$, where $v(n)$ is either $\tilde{\omega}(n)$ or $\tilde{\Omega}(n)$, the number of distinct prime factors and the total number of prime factors p of a positive integer n, such that $s_q(p) \equiv a \mod b$, $(a, b \in \mathbb{Z})$.

Jakub Krásenský
Czech Technical University in Prague

Number systems in imaginary quadratic fields

Abstract: We study a certain type of generalised number systems which allow unique representation of algebraic integers in a given number field: For a ring R, take any $\beta \in R$ and let $D \ni 0$ be a finite subset of R. We say that (β, D) is a *GNS* in R if every nonzero element of R has a unique representation of the form

$$x = \sum_{k=0}^{N} \beta^k a_k, \qquad \text{where } N \in \mathbb{N}_0,\ a_k \in D,\ a_N \neq 0.$$

The element β is called *radix* or *base* and D is the *alphabet*. It is clear that the alphabet must be a full residue system modulo β.

GNSs in imaginary quadratic fields have been studied among others by W. Penney, I. Kátai, J. Szabó, G. Steidl, W. Gilbert or A. Vince. By results of Imre Kátai, it is known that if $R = \mathcal{O}_K$ is the ring of integers in an imaginary quadratic number field K, then $\beta \in \mathcal{O}_K$ can serve as a radix of a GNS precisely if $|\beta| \neq 1$ and $|1 - \beta| \neq 1$. We extend this result also to all non-maximal imaginary quadratic orders (such as $\mathbb{Z}[\sqrt{3}\mathrm{i}]$) and, furthermore, characterise when it is possible to construct infinitely many GNSs with given radix β. This is joint work with A. Kovács.

Renan Laureti
Université de Lorraine

Dependances between the digits of β-expansions

Abstract: A common process in the study of normality to a given base is to consider the digits of the expansion of a real number in that base as random variables, in order to use results of the theory of probability. For example, one way to prove the theorem of Borel, which states that almost every real number (for Lebesgue measure) is absolutely normal, that is normal simultaneously to every integer base, is to use the law of large numbers. Another result that is used in effective algorithms of construction of normal numbers is Hoeffding's inequality, that allows to bound the amount of non (ε, k)-normal numbers in a given base.

However, these results can only be applied to bases where the random variables representing the digits are independant. This is not the case for β-expansions, and in this poster we will see more about the dependances of digits in those bases. We will then see how we can still apply versions of Hoeffding's inequality in some cases using a structure of Markov's chain of the random variables representing the β-expansions digits when they differentiated between the types of cylinders associated to those digits.

Nathan Toumi
IECL Lorraine

The level of distribution of the sum-of-digits function in arithmetic progressions

Abstract: The aim of our work is to generalize, for any base of numeration and a more general sequence than the Thue–Morse sequence, recent results obtained by L. Spiegelhoferregarding the level of distribution of sequences related to the sum-of-digits function. He showed that the Thue–Morse sequence has level of distribution of 1 improving on a former result of Fouvry and Mauduit. The heart of the proof for our generalization lies in an explicit estimate on Gowers norms, it follows the lines of Konieczny's proof. We make the calculations explicit to provide an explicit gain in the exponent in the main results.