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Periodicity and pure periodicity in alternate base systems

Abstract: Alternate base $\boldsymbol{\mathcal{B}}$ is given by a p-tuple $(\beta_1, \beta_2, \ldots, \beta_p)$ of real numbers greater than 1. We investigate in which cases all rational numbers $\frac{p}{q}$ in the interval $(0, 1)$ have an eventually periodic $\boldsymbol{\mathcal{B}}$-expansion. We show that this property forces the product $\delta = \beta_1\beta_2 \cdots \beta_p$ to be a Pisot or a Salem number. Analogic conclusion was earlier derived by Charlier, Cisternino and Kreczman, under a stronger requirement that $\frac{p}{q}$ has an eventually periodic expansion in every alternate base obtained by a cyclic shift of the original p-tuple.

We further examine under which circumstances there exists a $\gamma > 0$ such that every rational number in the interval $(0, \gamma)$ has a purely periodic $\boldsymbol{\mathcal{B}}$-expansion. We show that a necessary condition for this phenomenon is that δ is a Pisot or a Salem unit. We also provide a sufficient condition. We thus generalize the results known for the Rényi numeration system, i.e. for the case when $p = 1$, obtained by Schmidt, Akiyama, Adamczewski et al. and others. At the end, we present a class of alternate bases with $p = 2$, for which γ can be chosen to be 1.

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