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Numeration systems for aperiodic Wang tilings

Abstract: In this talk, we aim to explain how the study of small sets of aperiodic Wang tiles and their associated nonperiodic tilings naturally leads to the theory of numeration systems.

In one-dimension, one-sided automatic sequences can be constructed by feeding the representation of nonnegative integers into an deterministic automaton with output. Similar results hold in two dimensions for describing configurations of the whole plane covered by Wang tiles. The statement of such results needs numeration systems representing all integers regarless of the sign, as done for example by the two's complement numeration system.

We present a complement version of Dumont-Thomas numeration systems for $\mathbb{Z}$ based on any two-sided periodic point with growing seed of a substitution. As such, we recover the two's complement numeration system and its Fibonacci analog. These numeration systems can be used to describe configurations in self-similar Wang subshifts. This includes the metallic mean Wang shifts introduced earlier this year whose dynamical properties are related to the metallic mean numbers, that is, the positive root of $x^2 - nx - 1$ where $n \geq 1$ is an integer.

Surprisingly, another numeration system appear in the description of metallic mean Wang shifts, namely the balanced representation of real numbers which was already used by Kari and Culik in 1996 to show the existence of tilings with their aperiodic set of 14 and 13 tiles respectively.

Many of the results presented in the talk are joint work with Jana Lepšová who defended her Ph. D. thesis in May 2024.