Abstract: A β encoder (Daubechise et al., 2006) is an analog-to-digital (AD) converter based on β transformation. This AD converter was developed to overcome the drawback that AD conversion methods based on binary expansion are not robust to threshold variations. The goal of a β encoder is to obtain coefficients of β-expansion of the input analog value \( x \) with \( \beta \in (1, 2) \). A scale-adjusted β expansion is given by

\[
x = (\beta - 1) \sum_{i=1}^{\infty} a_i \beta^{-i}.
\]

There are uncountably many β expansions for a single \( x \). Let \( \nu_i \) denotes the threshold at the \( i \)-th iteration, allowing for fluctuations. We can model the process of β encoder as follows: With initial value \( x_0 = x \),

\[
a_i = Q_{\nu_i}(\beta x_{i-1}), \quad x_i = \beta x_{i-1} - a_i, \quad i \geq 1
\]

where \( Q_{\nu}(x) = 0 \) if \( x < \nu \) and \( Q_{\nu}(x) = 1 \) if \( x \geq \nu \). If \( \nu_i \in [1, 1/(\beta - 1)] \) is satisfied, the \( n \)-bit approximation error \(|x - (\beta - 1) \sum_{i=1}^{n} a_i \beta^{-i}| \) decreases exponentially in \( n \). Hence β encoder is robust to the fluctuation of the threshold.

The β-ary to binary conversion (Matsumura and Jitsumatsu, 2016) is a post-processing for a β encoder, which generates the binary expansion \( b_i \)'s of \( x \) whose scale-adjusted β expansion is \( a_i \).

Our central concern is how many bits of β expansion are needed to correctly determine the first \( n \) binary expansions of \( x \).

In this talk we discuss i) the approximation error of the proposed method, ii) the effect of mismatches in β values, and iii) the extension to the case \( \beta > 2 \).