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On β-ary to binary conversion from an engineering point of view

Abstract: A β encoder (Daubechise et al., 2006) is an analog-to-digital (AD) converter based on β transformation. This AD converter was developed to overcome the drawback that AD conversion methods based on binary expansion are not robust to threshold variations. The goal of a β encoder is to obtain coefficients of β-expansion of the input analog value x with $\beta \in (1, 2]$. A scale-adjusted β expansion is given by

$$x = (\beta - 1)\sum_{i=1}^{\infty} a_i \beta^{-i}.$$

There are uncountably many β expansions for a single x. Let ν_i denotes the threshold at the i-th iteration, allowing for fluctuations. We can model the process of β encoder as follows: With initial value $x_0 = x$,

$$a_i = Q_{\nu_i}(\beta x_{i-1}), \quad x_i = \beta x_{i-1} - a_i, \quad i \geq 1$$

where $Q_\nu(x) = 0$ if $x < \nu$ and $Q_\nu(x) = 1$ if $x \geq \nu$. If $\nu_i \in [1, 1/(\beta - 1)]$ is satisfied, the n-bit approximation error $|x - (\beta - 1)\sum_{i=1}^{n} a_i \beta^{-i}|$ decreases exponentially in n. Hence β encoder is robust to the fluctuation of the threshold.

The β-ary to binary conversion (Matsumura and Jitsumatsu, 2016) is a post-processing for a β encoder, which generates the binary expansion b_j's of x whose scale-adjusted β expansion is a_i. Our central concern is how many bits of β expansion are needed to correctly determine the first n binary expansions of x.

In this talk we discuss i) the approximation error of the proposed method, ii) the effect of mismatches in β values, and iii) the extension to the case $\beta > 2$.