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Analysis of Regular Sequences: Summatory Functions and Divide-and-Conquer Recurrences

Abstract: In simplest terms, a sequence x is called q-regular for some integer $q \geq 2$ if there are square matrices $A_0, \ldots, A_{q-1}$, a row vector u and a column vector w such that for all integers $n \geq 0$,

$$x(n) = uA_{n_0} \ldots A_{n_{\ell-1}} w$$

where $(n_{\ell-1}, \ldots, n_0)$ is the $q$ary expansion of n.

In the asymptotic analysis of regular sequences, it is usually advisable to study their summatory function because the original sequence has a too fluctuating behaviour. It turns out that the summatory function $N \mapsto \sum_{0 \leq n < N} x(n)$ of a q-regular sequence x has an asymptotic expansion

$$\sum_{0 \leq n < N} x(n) = \sum_{\substack{\lambda \in \sigma(C) \\ |\lambda| > R}} N^{\log_q \lambda} \sum_{0 \leq k < m_C(\lambda)} \frac{(\log N)^k}{k!} \, \Phi_{\lambda k}(\log_q N) + O\big(N^{\log_q R} (\log N)^{\kappa}\big)$$

as $N \to \infty$, where the $\Phi_{\lambda k}$ are suitable 1-periodic continuous functions and $\sigma(C)$, m_C, R, κ depend on the regular sequence.

It might be that the process of taking the summatory function has to be repeated if the sequence is fluctuating too much. In this talk we report on results that for all regular sequences except for some degenerate cases, repeating this process finitely many times leads to a “nice” asymptotic expansion containing periodic fluctuations whose Fourier coefficients can be computed using the results on the asymptotics of the summatory function of regular sequences.

In a recent paper, Hwang, Janson, and Tsai perform a thorough investigation of divide-and-conquer recurrences. These can be seen as 2-regular sequences. By considering them as the summatory function of their forward difference, the results on the asymptotics of the summatory function of regular sequences become applicable. We thoroughly investigate the case of a polynomial toll function.

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