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Numbers expressible by quotients or differences of two Pisot numbers

Abstract: In 1945, Salem himself proved that every Salem number is expressible as a quotient of two Pisot numbers. On the other hand, in 2004 the author showed that every positive algebraic number is a quotient of two Mahler measures. Recall that the *Mahler measure* $M(\alpha)$ of a nonzero algebraic number α is the modulus of the product of its conjugates lying outside the unit circle and the leading coefficient of its minimal polynomial in $\mathbb{Z}[x]$. Hence, for a real algebraic number $\alpha > 1$, we have $M(\alpha) = \alpha$ if and only if α is a Salem number or a Pisot number. The following theorem implies both these results:

Theorem 1. *Every real positive algebraic number α of degree d is expressible as a quotient of two Pisot numbers of degree d from the field $\mathbb{Q}(\alpha)$.*

Earlier, the author also investigated various sumsets and difference sets involving Salem and Pisot numbers. Now, we show that

Theorem 2. *Every Salem number is expressible as a difference of two Pisot numbers*

and study which other algebraic integers are expressible in this way.